here obtained. The value obtained by E. Rutherford* for the velocity of the negative ions produced in dry carbonic acid (0·78 cm. per second) by the action of ultra-violet light, is quite near to that here obtained (0·81 cm. per second) for the ions produced by Röntgen rays, but his values for dry air (1·4 cm. per second) and dry hydrogen (3·9 cm. per second) are considerably smaller.

In discharge from points, A. P. Chattock† has obtained for the velocities of the positive and negative ions in dry air 413 and 540 cm. per second respectively for a field of 1 E.S.U. per cm., which values are quite close to those obtained here for the ions produced by Röntgen rays.

J. S. Townsend‡ has shown that from the coefficients of diffusion of the ions and from their velocities it is possible to compare the charges carried by the different ions, and also to compare them with those carried by the ions in the electrolysis of liquids. By using the velocities given above with the coefficients of diffusion determined by J. S. Townsend, the values of N*e* are obtained, N being the number of molecules in 1 c.c. of the gas and *e* the charge carried by each ion. The results thus obtained for the moist gases, air, oxygen, and hydrogen, perhaps justify the statement that the charges carried by the positive and the negative ions are equal, and that the charge is the same for the different gases, and is equal to the charge carried by the hydrogen ion in the electrolysis of liquids.

The values of N*e* obtained for the positive ions in these gases when dry are considerably larger than the above, while in carbonic acid all of the results are over 20 per cent. smaller.

"Mathematical Contributions to the Theory of Evolution. VIII. On the Correlation of Characters not Quantitatively Measurable." By KARL PEARSON, F.R.S. Received February 7, —Read March 1, 1900.

(From the Department of Applied Mathematics, University College, London.)

(Abstract.)

1. In August last I presented to the Society a memoir on the inheritance of coat-colour in thoroughbred horses, and of eye-colour in man. This memoir, which was read in November of last year, presented the novel feature of determining correlation between characters which were not capable *à priori* of being quantitatively measured. The theoretical

* E. Rutherford, 'Camb. Phil. Soc. Proc.,' vol. 9, Part VIII.

† A. P. Chattock, 'Phil. Mag.,' November, 1899.

‡ J. S. Townsend, 'Phil. Trans.,' A, vol. 193, 1899.

part of that memoir was somewhat brief, but I showed by illustrations that the method could be extended to deal with problems like the effectiveness of vaccination and of the antitoxin treatment in diphtheria. More recently, in studying the phenomena of reversion in Basset Hounds, Mr. Bramley-Moore indicated to me how my method, although correct in theory, differed sensibly in the numerical results with the processes of interpolating employed. I then proposed a new method, and the analytical discussion of its details was worked out in part by Mr. Bramley-Moore himself, by Mr. L. N. G. Filon, M.A., and by myself. Dr. Alice Lee also came to our assistance, and the result is the present joint paper. On the basis of the new methods, we have already worked out upwards of sixty coefficients of correlation, principally of heredity. Thus the thirty-six coefficients of heredity for coat-colour in horses and eye-colour in man have been re-calculated, as well as twelve coefficients for heredity in coat-colour of Basset Hounds given in a paper on the Law of Reversion presented on December 28th, and about to appear in the 'Proceedings.' The great growth of the theoretical investigations has, however, compelled me to break up the old memoir* of last August into two parts, the one (the present) dealing only with theory, and the other with its application to inheritance in the horse and man.

2. The theory of the present memoir depends upon a very simple feature of normal correlation. If $z\delta x_1 \delta x_2 \ldots \delta x_n$ be the frequency of a complex of characters lying between x_1 and $x_1 + \delta x_1$, x_2 and $x_2 + \delta x_2 \ldots$, x_n and $x_n + \delta x_n$, where x_p is the deviation of the pth character from its mean, then

$$\frac{dz}{dr_{pq}} = \frac{d^2 z}{dx_p \, dx_q},$$

where r_{pq} is the correlation of the pth and qth organs.

This simple differential relation enables us to expand z for any number of characters in powers of the correlation coefficients (necessarily less than unity) by Maclaurin's theorem. But since we may replace a differential with regard to a coefficient of correlation by a double differential with regard to the corresponding organs, the coefficients of correlation may be put zero *before* instead of after the differentiation. In other words, we obtain a symbolic operator which, applied to a normal surface of frequency for n-uncorrelated organs, converts it into a correlated surface of frequency with $\frac{1}{2}n(n-1)$ coefficients of correlation of arbitrary values. This operator gives us by aid of certain symbolic equations the expansion of the n-fold integral

$$\int_{h_1}^{\infty} \int_{h_2}^{\infty} \int_{h_3}^{\infty} \ldots\ldots \int_{h_n}^{\infty} z \, dx_1 \, dx_2 \, dx_3 \ldots\ldots dx_n$$

* That memoir was at my own request returned for revision after being accepted for the 'Philosophical Transactions.'

in terms of the $\frac{1}{2}n(n-1)$ coefficients of correlation, and a series of new functions which we term the v-functions. These satisfy the difference equation:

$$v_n = xv_{n-1} - (n-1)v_{n-2}$$

and the differential equation

$$\frac{dv_n}{dx} = nv_{n-1}.$$

In fact

$$v_n = x^n - \frac{n(n-1)}{2\underline{|1}} x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2^2\underline{|2}} x^{n-4}$$
$$+ (-1)^r \frac{n(n-1) \ldots\ldots (n-2r+1)}{2^n\underline{|r}} x^{n-2r} + \ldots\ldots$$

The calculation of these functions is shown to be easy, and their properties are investigated. In this manner the volume of a frequency surface of the nth order cut off by n planes parallel to the n co-ordinate planes is shown to be capable of calculation, and its value is determined in the numerical illustrations given for example of 1, 2, 3 up to 6-fold correlation. It may be noted that by putting $z_1 = z_2 = z_3 = \ldots\ldots = z_n$, we have really obtained a result which enables us to find the "area" of a "spherical triangle" in n-fold hyperspace in terms of a series ascending by powers and products of the cosines of the angles between its faces.

The application of these results to the correlation of characters not quantitatively measurable, arises from the fact that the n-fold integral above given, and which we have shown how to evalute, measures the total frequency beyond certain boundaries. We can observe, for example, whether horses' coats are bay or darker (or chestnut or lighter), whether eyes are grey or lighter (or, dark grey or darker). Thus by forming mass frequencies instead of frequency distributions for small changes of character, we can find equations to determine the correlation. The probable error of such correlation, the convergency of the series, and other points are investigated.

3. Some discusssion is given to the problem of association, and coefficients allied to Mr. Yule's coefficient of association but somewhat closer in value to the coefficient of correlation are considered, and their relative closeness measured.

4. A number of illustrations of the new method are given from heredity in horses, dogs, and man, and it is shown how normality of frequency must even for such a character as stature* only be looked upon as a first approximation.

An investigation is also made into the influence of superior stock

* Cited by so many as an example of "normality."

in producing superior offspring. It is shown, for example, that if an individual who possesses a degree of character only found in one in twenty be considered "exceptional," then eighteen times as many exceptional men will be born of non-exceptional parents as of exceptional parents; but on the other hand, exceptional parents produce exceptional offspring at a rate ten times as great as non-exceptional parents, the greater gross product of the latter being due to their much greater numbers. In other words, distinguished parents are more likely to have distinguished offspring than undistinguished—ten times as likely—and yet only one distinguished man in nineteen will be born of distinguished parents. The importance of such conceptions for both natural and artificial breeding can hardly be over-estimated.

March 8, 1900.

The LORD LISTER, F.R.C.S., D.C.L., President, in the Chair.

A List of the Presents received was laid on the table, and thanks ordered for them.

The Bakerian Lecture, "The Specific Heat of Metals and the Relation of Specific Heat to Atomic Weight," was delivered by Professor W. A. Tilden, F.R.S.

BAKERIAN LECTURE.—"On the Specific Heat of Metals and the Relation of Specific Heat to Atomic Weight." By W. A. Tilden, D.Sc., F.R.S., Professor of Chemistry in the Royal College of Science, London. With an Appendix by Professor John Perry, F.R.S. Received February 9,—Read March 8, 1900.

(Abstract.)

The experiments described in this paper were begun with the object of assisting in the determination of the relative values of the atomic weights of cobalt and nickel, but were continued with the further purpose of testing the validity of the law of Dulong and Petit.

The metals cobalt and nickel closely resemble each other in density, melting point, and other physical properties, as well as in atomic weight. The pure metals were prepared for the purposes of these experiments with the most scrupulous care, the cobalt by taking